NOTATION

C_D = drag coefficient $\left(= \frac{4}{3} \frac{(\rho_s - \rho) gD}{\rho u^2}\right)$
D = diameter of sphere
du/dr = velocity gradient in fluid
g = gravitational acceleration
K = constant of proportionality
L = length of viscometer tube
N_{Re} = Reynolds number
P = pressure
p = an arbitrary normal pressure
Q = volume flow rate
R = radius of viscometer tube
u = velocity of solid particle through fluid

Greek Letters

η = coefficient of rigidity
ϕ = function
ρ = density of fluid
ρ_s = density of solid
τ = fluid shear stress
τ_y = plastic yield stress
θ = angle of normal to slip-surface in flow model

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Prediction of Pressure Drop for Two-Phase, Two-Component Concurrent Flow in Packed Beds

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Two-phase, gas-liquid concurrent flow in packed beds was investigated with the use of an air-water system and 2-, 4-, and 6-in. diameter columns packed with tabular alumina particles of 0.025 and 0.027 ft. diameters. Total pressure drop, column operating pressure, and liquid saturation were measured as functions of gas flow rate, fluid temperatures, and flow direction at several constant liquid flow rates for each column.

Correlation of the frictional pressure loss was achieved in terms of a defined two-phase friction factor and a second correlating parameter which is a function of the liquid and gas Reynolds numbers. A viscosity correction factor was required to extend the friction factor correlation to include liquid viscosities widely divergent from that of water.

The liquid saturation data for both upward and downward flow were correlated in terms of the ratio of mass flow rates of the respective phases.

The general field of multiphase flow has received much attention in recent years because of its widespread occurrence in engineering operations. It is encountered in such basic areas as distillation, evaporation, heat transfer, gas absorption, and other areas of the chemical processing industry. The most common type of multiphase flow involves gas and liquid phases, and the term two-phase flow will be taken as the gas-liquid combination in this paper.

Much effort has been expended for research on two-phase flow, although only a very minute portion of this research has been concerned with two-phase concurrent flow in packed beds, the reason for which was the apparent lack of a practical application for this work. However it has been shown (*1*) that under certain conditions, concurrent gas absorption is a more desirable operation than is gas absorption utilizing countercurrent flows. With countercurrent operation, flow rates are limited by the flooding point of the column, while the only limit to flow rates in concurrent operation is the amount of power to be expended in forcing the fluids through the column, thus providing a

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much more flexible design within which to optimize equipment and reduce costs (*1*).

Since the flow rates for concurrent flow are restricted only by the allowable pressure drop through the bed, rather than by a density difference, a very wide range of throughputs are possible (*6*) for which correlation and design information are scarce. The two correlations published to date (*5*, *11*) have been based on the classic Lockhart-Martinelli correlation (7 to 9) for two-phase flow in open conduits.

Except for the literature on two-phase flow in porous media, only four references were available pertaining to two-phase concurrent flow in packed beds. The desirability of concurrent flow, from the standpoint of tower pressure drop, was noted by Piret et al. (*10*) in an early work in which they reported the pressure drop encountered in countercurrent flow to be almost double that encountered in countercurrent flow of air-water mixtures. The next reference was that of Dodds et al. (*2*), who presented pressure drop data for two-phase, vertically downward concurrent flow, but a general correlation was not attempted.

A much wider range of experimental variables was covered by Larkins (*4*), who studied the vertical downward flow of several gas-liquid systems through packed beds. A Martinelli type of two-phase parameter was utilized for correlation of his data. The data of Weekman and Myers (*11*) for downward concurrent flow were also correlated in terms of Lockhart-Martinelli two-phase parameters. In this work it is assumed that the packing supports essentially all of the liquid and hence that a static correction is not necessary for liquid loadings below 25,000 lb./(sq. ft.)(hr.). Thus the total measured pressure drop is identical to the frictional pressure drop by this analysis. As noted, each of these investigators collected data only for downward flow and postulated that the pressure drop for two-phase concurrent flow through packed was independent of flow orientation.

There were several objectives of this investigation. First, it was desired to develop a mathematical model of two-phase concurrent flow through packed beds which would provide a basis for the correlation of the experimental data. The second was to obtain experimental data for both upward and downward flow in order either to verify or dispute the assumption made by the previous investigators of the independence of pressure drop and flow orientation. The final objectives were to derive correlations and to present calculation procedures which enable prediction of the pressure drop for two-phase concurrent flow for use in the design of packed columns and auxiliary equipment.

EXPERIMENTAL PROGRAM

In general terms the experimental program consisted of obtaining sufficient data to establish suitable correlations for the prediction of frictional pressure drop in terms of the system variables for both upward and downward vertical two-phase concurrent flow in a packed bed. Specifically, this required measurement of column pressures and pressure drops, fluid temperatures, and liquid saturations over a range of liquid and gas flow rates for several different packing diameter and column diameter combinations.

The equipment used in the investigation consisted of 2-, 4-, and 6-in. diameter, 84-in. long packed columns and the related components and piping required to establish and measure the above quantities. An air-water system was used and the packing material was tabular alumina of 0.027 and 0.025 ft. diameters. A schematic diagram of the experimental apparatus is shown in Figure 1.

The columns were of transparent Busada 210 butyrate plastic tubing. Quick-closing valves were attached at the top and bottom of each column, and their respective handles were joined rigidly by a metal rod. This provided for simultaneous closing of the valves and thus enabled determination of the liquid saturation. A retaining screen was attached inside each valve immediately adjacent to the valve seat and the valves themselves were packed with the tabular alumina.

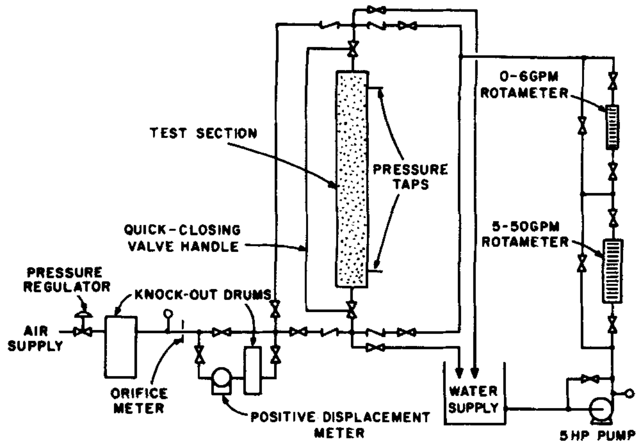

Fig. 1. Schematic diagram of experimental apparatus.

Pressure taps were drilled 6 in. from both top and bottom of each column and a second hole was drilled 2 in. below each respective pressure tap. These were fitted with separators which returned the liquid to the column through the lower tap and thus maintained the manometer leads in a single-phase gas condition.

The columns were packed with the use of procedures which were designed to obtain reproducible bed properties. The lower valve of the column was closed and the column was partially filled with a known quantity of water. A given volume of packing material was weighed and dumped into the column while the sides of the column were tapped with a rubber hammer to ensure complete settling of the particles. The liquid level was noted and recorded after the addition of the given quantity of packing material, and the bed porosity was calculated from these measurements.

Prior to making two-phase determinations, single-phase data were taken for each fluid over a range of flows for use in single-phase correlations and as a check of the operating procedures. Data were then taken for a range of gas flow rates for each of several constant liquid flow rates. For each experimental determination, the following data were recorded: the flow rate of each stream, the temperature of each stream, the column pressure and pressure drop, plus a note concerning the observed flow pattern in the packed bed. To determine the liquid saturation, the quick-closing valves were shut simultaneously by the common valve handle. After waiting 10 min. for the liquid to drain, we measured the height of the liquid in the column. The column pressure after shut-in was also recorded. After completion of the experimental determination, moving pictures were made of representative flow patterns in each packed bed.

DESCRIPTION OF OBSERVED FLOW PATTERNS

Three distinct flow patterns were observed experimentally. These were termed bubble flow, slug flow, and spray flow, and the relative location of each of these flow regimes in terms of the mass velocities of the respective phases is given in Figure 2. In order to visualize each of these flow patterns, a given liquid rate will be discussed as the gas rate is varied from zero to its maximum value. With single-phase liquid flow established in the column, the bubble blow regime is encountered with the introduction of very low gas flows. This regime is characterized by bubbles of gas flowing unbroken in the liquid continuous phase at slightly higher velocities than the liquid phase. As expected, the higher the given liquid rate, the wider the range of gas flows which will produce bubble flow.

As the gas rate is increased further at the given liquid rate, a nonhomogeneous flow regime termed slug flow is

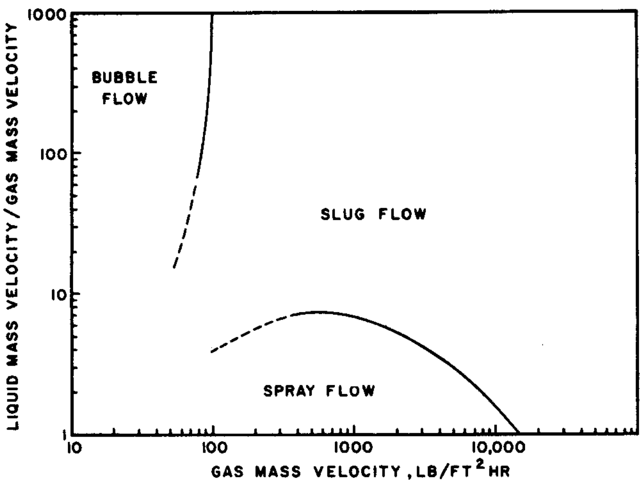

Fig. 2. Flow regimes, vertical flow.

encountered. This regime is characterized by alternate portions of more dense and less dense mixtures of the two phases passing through the column. At the onset of slugging, a portion of the mixture with a density approaching that of the liquid collects at the entrance to the column and is swept through the column by an alternate portion of the mixture with a density approaching that of the gas phase. As the slugs first occurred, they appeared to be 4 to 6 in. thick separated by approximately 12 in. of the lighter phase, which propelled them through the packed bed at a velocity of approximately 6 ft./sec. At low liquid rates there were roughly 30 slugs/min., increasing to nearly twice that figure at higher flows. As the gas flow was increased further for the given liquid rate, the frequency of the slugs increased, and the difference in density between the alternate slugs of fluid became progressively less.

With further increases in gas flow rate the density difference between the alternate slugs disappeared entirely, producing the third flow pattern, spray flow. This is a gas continuous flow regime with the liquid being carried through the column suspended as a heavy mist in the gas stream. At this point the packing surfaces were covered by a thick layer of liquid which became progressively thinner with increasing gas rate. At the upper limiting gas rates for the lower liquid rates, this liquid layer on the packing became very thin and essentially all the liquid was transported in the gas stream as a very fine mist.

It should be noted that the lines drawn on Figure 2 to separate the three flow regimes are actually transition regions rather than points of abrupt change from one flow type to another. Either of the flow patterns may be encountered in the vicinity of this separating line which was drawn to locate only qualitatively the various flow regimes. Figure 2 is applicable for both upward and downward flow with the only differences being that slugging is initiated at slightly lower gas velocities and persists to slightly higher gas velocities for upward flow at a given liquid rate.

It was noted that there was no abrupt change of pressure drop with gas mass flow rate for any of the transitions from one flow type to another. There were also no abrupt changes noted in the liquid saturation data with the observed flow pattern. This suggests that the correlation of the data should be possible, independent of the flow pattern.

CORRELATION OF EXPERIMENTAL DATA

To achieve a basis for generalized correlations of the experimental data, momentum balances were written for each phase with the assumption of a separated flow mathematical model (*3*). Addition and further manipulation of the momentum balances resulted in the total pressure drop being expressed in terms of the sum of the frictional pressure drop, the static pressure drop, and the pressure drop due to acceleration of the fluids:

$$-(P_2 - P_1) = \Delta P_{TPf} + (\cos\,\theta/\overline{A})\,(C_2\overline{A}_G L + \rho_L \overline{A}_L L + C_3\overline{A}_G L^2/2) + (W_G/g_c A)\,(V_{G_2} - V_{G_1}) + (W_L/g_c A)\,(V_{L_2} - V_{L_1}) \quad (1)$$

Assumptions made in the derivation of Equation (1) were a linear variation of pressure and a linear variation of $\overline{A}_G$. It is noted that for horizontal flow, $\theta = 90$ deg., $\cos\theta = 0$, and the static term drops out. Preliminary calculations indicated that the pressure loss due to acceleration of the fluids was negligible below operating pressures of 50 lb./sq. in. gauge. Rewriting Equation (1) for vertical flow and neglecting ΔP_{acc}, we obtain

$$-(P_2 - P_1) = \Delta P_{TPf} + (1/A)\,(C_2\overline{A}_G L + \rho_L \overline{A}_L L + C_3\overline{A}_G L^2/2) \quad (2)$$

The use of Equation (2) requires evaluation of the two-phase frictional pressure drop ΔP_{TPf} and $\overline{A}_L$ (and thus $\overline{A}_G$). A knowledge of the liquid saturation R_v is tantamount to a knowledge of $\overline{A}_L$, and the two-phase frictional pressure drop may be expressed in terms of a two-phase friction factor f_{TPf}, which is defined below. Thus correlations of the quantities f_{TPf} and R_v in terms of known system variables for both upward and downward flow are desired, and it is the purpose of this section to establish such correlating relationships for each of these quantities.

The two-phase friction factor to be employed was defined in the manner of a single-phase friction factor:

$$f_{TPf} = \frac{(\Delta P/L)_{TPf}\, D_e\, g_c}{2\rho_{G1}\overline{V}_{GS}^2} \quad (3)$$

Individual quantities of this defining equation require clarification. The two-phase frictional pressure gradient is the frictional pressure drop divided by the length of the test section. Gas density ρ_{G1} is the density at the entering temperature and pressure. The superficial gas velocity $\overline{V}_{GS}$ is the velocity which the gas would have if it were flowing in single-phase flow through the unpacked conduit at the entering density ρ_{G1}. The diameter D_e is an equivalent diameter which is four times the hydraulic radius for the packed bed. To determine the hydraulic radius R_H, the bed is assumed to be a number of parallel channels, the total volume of which is the volume of voids in the bed, and the total surface of which is the total surface area of the solids. With these assumptions, the hydraulic radius is then the ratio of the total volume of voids to the total area of solids. The equivalent diameter is then given by

$$D_e = 4R_H = \frac{\varepsilon}{1-\varepsilon}\,\frac{V_p}{S_p} \quad (3a)$$

For spherical particles

$$V_p/S_p = D_p/6 \quad (3b)$$

and

$$D_e = (2/3)\,(D_p)\left(\frac{\varepsilon}{1-\varepsilon}\right) \quad (3c)$$

The results of a dimensional analysis of two-phase flow with subsequent simplification reveal that the frictional

TABLE 1. CALCULATED EXPONENTS

Exponent	Upflow	Downflow	Average
a	0.761	0.773	0.767
b	−1.177	−1.157	−1.167
d	0.24	0.24	0.24

pressure gradient is given by

$$\left(\frac{\Delta P}{L}\right)_{TPf} = \frac{\overline{V}_{Gs}^2\,\rho_{G1}}{D_e\ g_c}\ \phi\,[(N_{Re\,L})^a (N_{Re_G})^b] \tag{4}$$

where both gas and liquid Reynolds numbers are defined in terms of the packing diameter D_p and the mass velocity for each of the respective Reynolds numbers is based on the total cross-sectional area of the unpacked conduit. The gas density to be used is again the entering density, ρ_{G1}. Rearranging Equation (4) and comparing with the definition of the two-phase friction factor given by Equation (3): we get

$$f_{TPf} = \frac{(\Delta P/L)_{TPf}\, D_e\, g_c}{2\overline{V}_{G_s}^2\,\rho_{G_1}} = 1/2\ \phi\,[(N_{Re_L})^a (N_{Re_G})^b] \tag{5}$$

or

$$f_{TPf} = \phi_1\,[(N_{Re_L})^a (N_{Re_G})^b] \tag{6}$$

for upward flow, and

$$f_{TPf} = \phi_2\,[(N_{Re_L})^a (N_{Re_G})^b] \tag{7}$$

for downward flow.

The remaining correlating relationships to be established are those for liquid saturation R_v. From a study of the graphs of in-place ratio vs. flowing ratio, the following equations were proposed:

$$R_v = \psi_1 (G_L/G_G)^d \tag{8}$$

$$R_v = \psi_2 (G_L/G_G)^d \tag{9}$$

for upward and downward flow, respectively.

It was assumed in Equations (6) through (9) that different functional forms were required to represent upward and downward flow, respectively, but that the values of the exponents a, b, and d would be identical for upward and downward flow. Experimental data and least-square techniques were used for evaluation of these exponents. The calculated values for each are given in Table 1.

Since the upflow and downflow values differed by less than 2% for each of the calculated exponents, average values were used. Substitution of the respective average values of the exponents into Equations (6), (7), (8), and (9) establish the proposed correlating relationships and evaluation of their respective functional form remains.

As the first step in the presentation of the correlated data, values of

$$1/Z = N_{Re_L}^{0.767} N_{Re_G}^{-1.167} \tag{10}$$

were calculated for each experimental determination. It was noted that the two-phase friction factor was an increasing function of $1/Z$ for both upward and downward flow. Therefore in order to produce a correlation with the general appearance of the standard friction factor–Reynolds number plot, the group $1/Z$ was inverted:

$$Z = \frac{N_{Re_G}^{1.167}}{N_{Re_L}^{0.767}} \tag{11}$$

The final plots of the two-phase friction factor f_{TPf} vs. the group Z are presented in Figures 3 and 4 for upward and downward flow, respectively. It was necessary for these to be plotted on logarithmic scales because of the wide range of values covered by each of the variables. The equations of the best fit for these respective data plots were determined with a computer and a curve-fitting program which calculated the least-square fit for any number of parameters and then indicated the degree of polynomial which would provide the minimum variance.

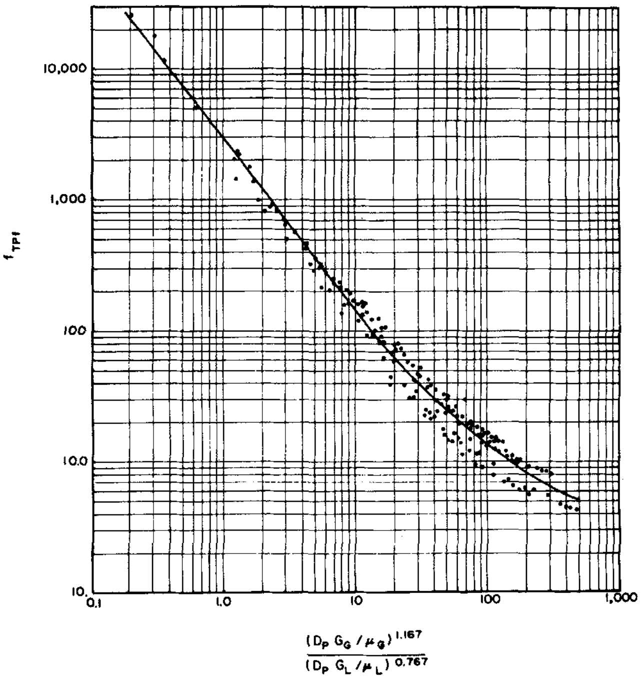

Fig. 3. Friction factor correlation, upward flow.

The equation for the upward flow data is

$$\ln f_{TPf} = 8.0 - 1.12\,(\ln Z) - 0.0769\,(\ln Z)^2 + 0.0152\,(\ln Z)^3 \tag{12}$$

$$(0.3 \le Z \le 500)$$

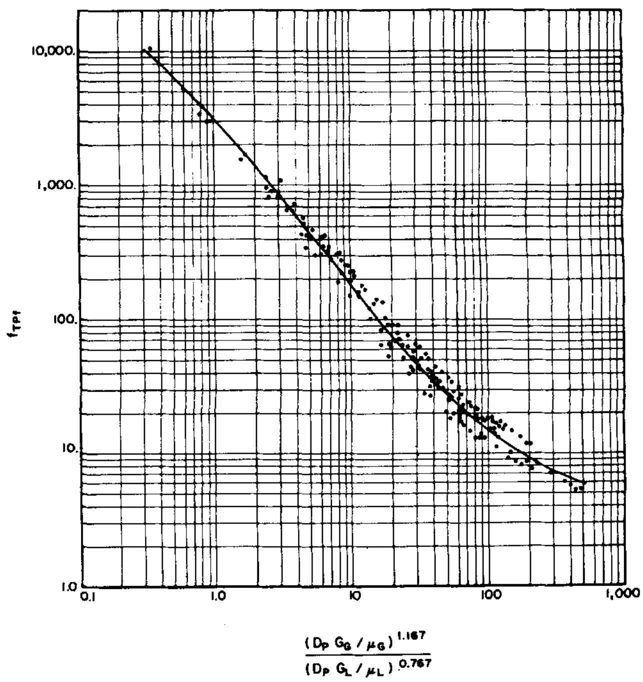

Fig. 4. Friction factor correlation, downward flow.

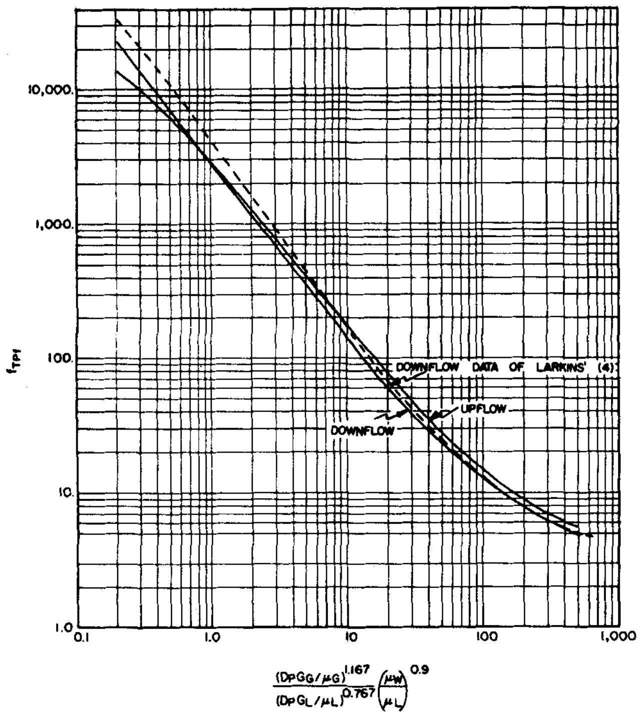

Fig. 5. Comparison of friction factor correlations.

and the curve representing this equation is reproduced in Figure 3. Rather than f_{TPf} and Z, it was necessary to use the logarithms of these quantities in Equation (12), because the data were presented on a logarithmic rather than a rectangular plot.

The equation for the downward flow data is

$$\ln f_{TPf} = 7.96 - 1.34 (\ln Z) + 0.0021 (\ln Z)^2 + 0.0078 (\ln Z)^3 \quad (13)$$
$$(0.2 \leq Z \leq 500)$$

The curve representing Equation (13) is reproduced in Figure 4.

For comparison, values of f_{TPf} and Z were calculated from the downward flow data of Larkins (4). The equation for these data is

$$\ln f_{TPf} = 8.37 - 1.372 (\ln Z') - 0.0315 (\ln Z')^2 + 0.0123 (\ln Z')^3 \quad (14)$$
$$(0.2 \leq Z' \leq 600)$$

where Z' differs from Z by a viscosity correction factor which will be discussed later. Equation (14) includes all of the Larkins data for spherical packing material and for Raschig ring packing material, but it does not include his data with 1/8-in. cylinders as packing material. A graphical comparison of Equations (12), (13), and (14) is presented in Figure 5.

Correlations of the liquid saturation data were obtained with the use of the exponent d reported in Table 1 in Equations (8) and (9). Values of $(G_L/G_G)^{0.24}$ were calculated for each experimental determination, and these were plotted vs. their respective liquid saturations. These results are shown graphically in Figure 6.

The curve-fitting computer program was applied to these data. The best fitting curve for the upward flow data was found to be a fourth-degree polynomial, but it was observed that this curve was closely approximated by the linear least-square curve:

$$R_\nu = -0.035 + 0.182 (G_L/G_G)^{0.24} \quad (15)$$
$$(1.0 \leq (G_L/G_G)^{0.24} \leq 6.0)$$

Therefore, because Equation (15) is much the easier equation with which to work, it will be utilized as the correlating equation. Although the data were quite scattered, essentially all data points were within ±25% of Equation (15).

For the downward flow data the best fitting curve was again a fourth-degree polynomial which was closely approximated by the linear least-square curve:

$$R_\nu = -0.017 + 0.132(G_L/G_G)^{0.24} \quad (16)$$
$$(1.0 \leq (G_L/G_G)^{0.24} \leq 6.0)$$

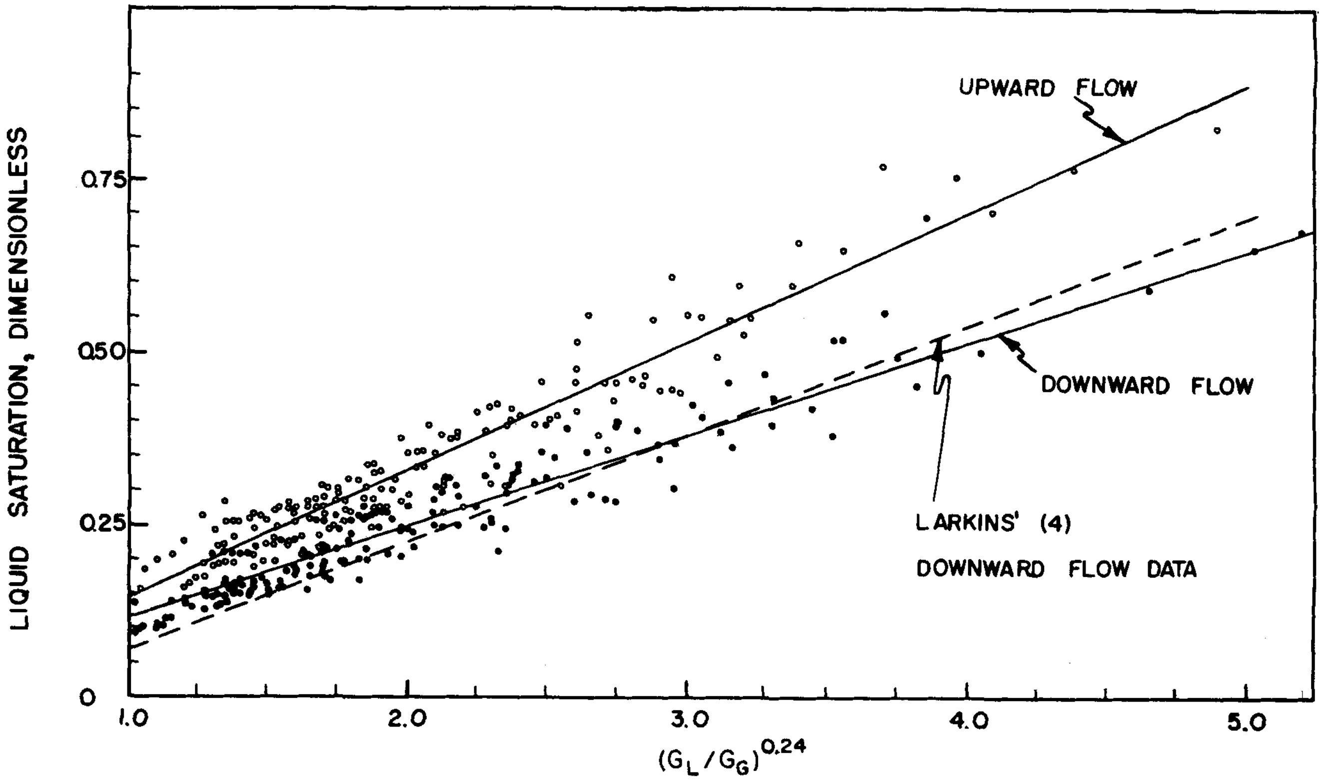

Fig. 6. Comparison of liquid saturation correlations.

Approximately 95% of the downflow data was within ±25% of Equation (16).

The correlating method was applied to the Larkins (4) downflow liquid saturation data. The minimum variance curve was a cubic equation, but once again the higher order equation was closely approximated by the linear least-square fit of the data:

$$R_v = -0.082 + 0.154(G_L/G_G)^{0.24} \quad (1.0 \leq (G_L/G_G)^{0.24} \leq 6.0) \tag{17}$$

A graphical comparison of Equations (15), (16), and (17) is given in Figure 6.

DISCUSSION OF RESULTS

Friction Factor Correlations

That the two-phase friction factor method of correlation is satisfactory is verified by Figures 3 and 4 for a wide range (10^3) of the correlating variable Z. As a further check of the validity of these correlations, the end points of each were checked with single-phase experimental data. It was found that each of the correlations merges into a single-phase gas correlation for higher values of Z.

The value of Z was calculated for single-phase gas flow for a number of experimental observations by taking the liquid Reynolds number as 1. Single-phase Z values calculated in this manner ranged from 155 to 5,000, with more than 75% of these calculated points falling within ±25% of the extended curves of Figures 4 and 5. However, this merging of the two-phase curve smoothly into the single-phase curve is to be expected because of the manner in which the two-phase friction factors were defined, that is, in terms of the superficial gas velocity. Because of the large magnitudes of the superficial gas velocity in this region of the curve, both f_{TPf} and Z are strongly influenced by this quantity, and the correlation does not change appreciably in passing from two-phase flow with a very small liquid-to-gas ratio to single-phase gas flow.

Single-phase liquid data could not be fitted to Figures 3 and 4 because of the presence of the superficial gas velocity factor in the defining equation for f_{TPf}. However it was possible to retain the correlation for gas rates very near zero. The experimental points for Z values between 0.1 and 1.0 are very low gas flows and correspondingly high liquid-to-gas ratios.

To provide a check of these friction factor correlations for a much wider range of experimental conditions than was employed in this investigation, the experimental data of Larkins (4) were converted to $f_{TPf} - Z'$ values for comparison with the data of Figures 3 and 4. This graphical comparison is included as Figure 5.

It is noted that the abscissa of Figure 5 differs from those of Figures 3 and 4 by a viscosity correction term $(\mu_{water}/\mu_L)^{0.90}$. It was necessary to include this factor in order to correlate the data for liquids of viscosity substantially different from that of water with the use of the derived correlating method. Successful correlation of these data, including liquid viscosities of 0.8 to 19.0 centipoise, was accomplished using this correction factor.

From Figure 5 it is seen that the two sets of downflow data approximate each other for large values of Z', while differences of 30 to 50% are encountered for small values of Z'. Part of this is attributed to the wide range of experimental variables employed, notably fluid properties and packing materials. In addition to the use of near-spherical packing material as was used in this investigation, 3/8-in. Raschig rings with maximum porosities of 52% were utilized by Larkings. Concerning the comparison of upward flow and downward flow as given by Figure 5, differences of only 0 to 25% between the two are exhibited over the entire range of Z' values, except at the very lowest values of Z'.

Liquid Saturation Correlations

The liquid saturation correlations given by Figure 6 exhibit quite a large scatter of the data. This is characteristic of liquid saturation data, however, with large variations having been reported by previous investigators in two-phase flow. Again, the proposed correlation was checked with downflow data from reference (4), and this graphical comparison is given in Figure 6. Maximum deviations between the two downflow correlations of approximately 35% were found for small values of (G_L/G_G).

It was noted previously that although fourth-power polynomials provided a better fit of the experimental data, linear equations would be utilized to represent the liquid saturation data. Considerable simplification of calculations is attained by this substitution, while no significant loss of accuracy is introduced. One limitation of the correlating equations is that neither passes through the origin although physically each must do so. For this reason, use of Equations (15) and (16) below a value of unity for $(G_L/G_G)^{0.24}$ is not recommended.

Scope of the Correlations

The correlations will be examined to ascertain their reliability. and their range of applicability. To establish their reliability, 318 individual two-phase data points were used with half of these for upflow and half for downflow. Thus each individual correlation is the result of approximately 160 data points distributed more or less equally over the range of the correlation. Besides these 318 data points, an additional 176 experimental data points from other sources were utilized as a check of the derived correlations.

To establish the range of applicability of the proposed correlations, the range of the experimental variables is considered. A wide variation in the flow rate of each phase was utilized with the gas flow rate extending from about 16 to 4,800 lb./(sq. ft.)(hr.) and the liquid flow rate having a range of 4,800 to 40,000 lb./(sq. ft.)(hr.). Use of the correlations for gas flow rates below 16 lb./(sq. ft.)(hr.) is not recommended. Only low column operating pressures were utilized, with the maximum being near 50 lb./sq. in. abs., and the use of the correlations much beyond this value is not recommended.

Gas viscosity was very nearly constant for the investigation at 0.018 centipoise. This however is not a serious limitation as the viscosities of most gases at moderate temperatures do not vary greatly. All data were obtained with liquid viscosities near 1 centipoise, and the experimental correlations are based only on these data. It is noted that with the viscosity correction factor $(\mu_w/\mu_L)^{0.9}$ utilized, data are correlated in Figure 5 with liquid viscosities ranging up to 19 centipoise. However it is suggested that caution be used in application of the Figure 5 correlations to systems having a liquid viscosity widely divergent from 1 centipoise.

The ratio of the fluid mass flow rates is the basis for the liquid saturation correlations. With the range of mass flow rates given above, it is seen that the ratio of these varies from 1 to 2,500. This range is reduced to approximately 1 to 6.5 in terms of the correlating group $(G_L/G_G)^{0.24}$, and the correlations should be adequate over this interval.

In addition to being a function of the mass flow ratio, the liquid saturation is also a complicated, although not a strong, function of bed porosity. Although the correlations of this investigation were derived with the use of bed porosities near 35%, it is believed that they may be used safely up to bed porosities of 50% because of the close agreement with the 52% bed porosity data (4) as shown by Figure 6.

The derived correlations incorporate the single-phase pressure loss prediction to give simple integrated relationships for two-phase pressure loss and are without means for adjusting for particle shape or unusual stacking patterns. It is thus recommended that their use be restricted to near-spherical particles.

Use of the Experimental Correlations

For design purposes it is assumed that the physical properties and dimensions of the bed, the column orientation, the flow rate of each phase, the fluid physical properties, and the delivered pressure of each phase are known. Unknown are the average pressure of the column and the total pressure drop through the bed which is the required quantity.

The frictional pressure drop may be obtained directly. From the known design variables, the value of the correlating group Z' is calculated, and the two-phase friction factor may be determined from Figure 5 or from Equation (12) or (13). The frictional pressure drop is then determined with a rearrangement of Equation (3) and the two-phase friction factor.

From this point a trial-and-error solution must be used, because, in addition to the total pressure drop, the average operating pressure is an unknown. An operating pressure is assumed and the total pressure drop is determined with Equation (2). This procedure is repeated until the average pressure, as determined from the calculated pressure drop, is sufficiently close to the assumed operating pressure.

Use of Equation (2) requires evaluation of $\overline{A}_L$ and $\overline{A}_G$. These are obtained from the derived liquid saturation correlations. A value of $(G_L/G_G)^{0.24}$ is calculated from the known quantities and Figure 6 or from Equation (15) or (16), depending upon whether the flow is upward or downward.

Use of Equation (2) is not recommended above column operating pressures of 50 lb./sq. in. abs. or for pressure drops greater than 40 lb./sq. in. abs. Beyond these conditions acceleration of the fluids may become a significant factor. It then becomes necessary to utilize Equation (1) which requires a knowledge of the initial and final velocities in addition to the other known quantities.

CONCLUSIONS

The conclusions drawn from this investigation of two-phase concurrent flow in packed beds are:

1. Correlation of the liquid saturation data for both upward and downward flow is achieved in terms of a function of the ratio of the mass flow rate of liquid to gas.
2. Correlation of the frictional pressure loss is achieved in terms of a defined two-phase friction factor and a correlating parameter Z which is a function of the liquid and gas Reynolds numbers.
3. A viscosity correction factor is required to extend the friction factor correlation to include liquid viscosities widely divergent from that of water. The reliability of the extended correlation is not determined.
4. The frictional pressure loss may be assumed to be independent of column orientation.
5. The pressure loss due to acceleration of the fluids is negligible for operating pressures below 50 lb./sq. in. abs.
6. The frictional pressure loss is independent of the two-phase flow pattern.

NOTATION

A = area of flow
$\overline{A}$ = average area
a, b, d = exponents of correlating groups
C_1 = constant $(=MW/RT)$
C_2 = constant $(=C_1P_1)$
C_3 = constant $(=C_1k)$
D_p = diameter of packing particle
D_e = equivalent diameter as defined by Equation (3a)
f = friction factor
G = mass velocity, based on the total cross-sectional area of the unpacked conduit
g_c = gravitational constant
k = constant, variation of pressure with y
L = length of packed section
N_{Re} = Reynolds number
N_{Re_G} = Reynolds number for gas phase $(=D_pG_G/\mu_G)$. The diameter is the particle diameter and the mass velocity is based on the total cross-sectional area of the unpacked conduit.
N_{Re_L} = Reynolds number for liquid phase $(=D_pG_L/\mu_L)$. The diameter is the particle diameter and the mass velocity is based on the total cross-sectional area of the unpacked conduit
P = pressure
R_v = liquid saturation, fraction of voids filled with liquid
R_H = hydraulic radius
S_p = surface area of packing particle
V = linear velocity
V_p = volume of packing particle
W = weight flow rate
y = linear distance through packed bed
Z = correlating parameter
Z' = correlating parameter

Greek Letters

ε = packed-bed porosity
θ = angle of inclination of the packed column from vertical
μ = viscosity
ρ = density
ϕ = function of
ψ = a function of

Subscripts

acc = acceleration
f = friction
G = gas
L = liquid
p = particle
s = superficial velocity, based on total area of unpacked column
TP = two phase
w = water
1 = upstream datum point
2 = downstream datum point

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